Fuzzy description logics and ontology

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Outline

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Motivation

- Semantic Web, in which ontologies play a key role, has been a hot “topic”.
- Recommended description language for ontology:
    - OWL full: undecidable.
    - OWL DL: based on $\mathcal{SHIQ}$ DL. In fact, it is equivalent to $\mathcal{SHOIN}(\mathrm{D})$.
    - OWL Lite: based on $\mathcal{SHIF}$ DL.
- However, $\mathcal{SHIQ}$ and $\mathcal{SHIF}$ DLs are not able to represent imprecise concepts, e.g. Young_People, Fast, and so on.
- Extending DLs by fuzzy set theory: deal with vagueness and imprecision. Start with fuzzy $\mathcal{ALC}$.

Fuzzy $\mathcal{ALC}$

Interpretation:

$$\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$$
$$C^{\mathcal{I}} : \Delta^{\mathcal{I}} \to [0,1]$$
$$R^{\mathcal{I}} : \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \to [0,1]$$

Concepts:

Syntax		Semantics	
$C, D \to$	$\top \mid$	$\top^{\mathcal{I}}(x)$	$= 1$
	$\bot \mid$	$\bot^{\mathcal{I}}(x)$	$= 0$
	$A \mid$	$A^{\mathcal{I}}(x)$	$\in [0,1]$
	$C \sqcap D \mid$	$(C \sqcap D)^{\mathcal{I}}(x)$	$= C^{\mathcal{I}}(x) \wedge D^{\mathcal{I}}(x)$
	$C \sqcup D \mid$	$(C \sqcup D)^{\mathcal{I}}(x)$	$= C^{\mathcal{I}}(x) \vee D^{\mathcal{I}}(x)$
	$\neg C \mid$	$(\neg C)^{\mathcal{I}}(x)$	$= \neg C^{\mathcal{I}}(x)$
	$\exists R.C \mid$	$(\exists R.C)^{\mathcal{I}}(x)$	$= \sup_{y \in \Delta^{\mathcal{I}}} R^{\mathcal{I}}(x,y) \wedge C^{\mathcal{I}}(y)$
	$\forall R.C$	$(\forall R.C)^{\mathcal{I}}(x)$	$= \inf_{y \in \Delta^{\mathcal{I}}} R^{\mathcal{I}}(x,y) \to C^{\mathcal{I}}(y)$

where $\wedge$:t-norm, $\vee$:t-conorm, $\neg$:negation, $\to$:implication

In fuzzy $\mathcal{ALC}$: $\wedge : \min, \vee : \max, \neg x \equiv 1-x, x \to y \equiv \max(1-x, y)$

Assertions: $\langle a : C \circ n \rangle, \mathcal{I} \models \langle a : C \geq n \rangle$ iff $C^{\mathcal{I}}(a^{\mathcal{I}}) \geq n$ (similarly for other relations and roles).

Terminological axioms: $A = C$ or $A \sqsubseteq C$

$\mathcal{I} \models A \sqsubseteq C$ iff $\forall x \in \Delta^{\mathcal{I}}. A^{\mathcal{I}}(x) \leq C^{\mathcal{I}}(x)$.

Decision problems

- **Satisfiability:** is there any model $\mathcal{I}$ of given $\mathcal{K}$?
- **Entailment:** given $\mathcal{K}$ and $\langle \alpha \circ n \rangle$, $\mathcal{K} \models \langle \alpha \circ n \rangle$?
- **Subsumption:** given $\mathcal{K}$, $\mathcal{K} \models C \sqsubseteq D$?
- **Best Truth Value Bound (BTVB):** given K,
  $glb(\mathcal{K}, a : C) = \sup\{n \mid \mathcal{K} \models \langle a : C \geq n \rangle\} =?.$

Tableau for satisfiability problem: consists of propagation rules.

Rules (excerpt):

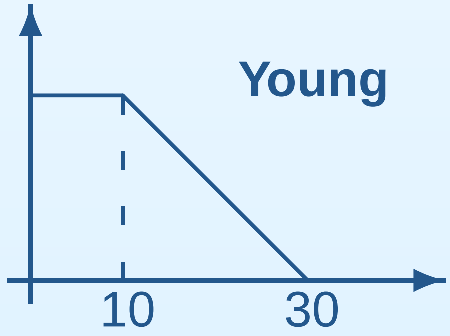
$(\neg_{\geq})$	$\langle w : \neg C \geq n \rangle \rightarrow \langle w : C \leq 1 - n \rangle$
$(\sqcap_{\geq})$	$\langle w : C \sqcap D \geq n \rangle \rightarrow$ $\langle w : C \geq n \rangle, \langle w : D \geq n \rangle$
$(\sqcup_{\geq})$	$\langle w : C \sqcup D \geq n \rangle \rightarrow$ $\langle w : C \geq n \rangle \mid \langle w : D \geq n \rangle$
$(\forall_{\geq})$	$\langle w_1 : \forall R.C \geq n \rangle, \psi \rightarrow$ $\langle w_2 : C \geq n \rangle$ if ψ is conjugated to $\langle (w_1, w_2) : R \leq 1 - n \rangle$
$\ldots$	$\ldots$

Example:

(1)	$\langle i : \exists about.(Car \sqcap Ferrari) \geq 0.6 \rangle$	
(2)	$\langle i : \exists about.Car < 0.6 \rangle$	
(3)	$\langle (i, x) : about \geq 0.6 \rangle$ $\langle x : (Car \sqcap Ferrari) \geq 0.6 \rangle$	$\exists_{\geq}$:(1)
(4)	$\langle x : Car \geq 0.6 \rangle$ $\langle x : Ferrari \geq 0.6 \rangle$	$\sqcap_{\geq}$:(3)
(5)	$\langle x : Car < 0.6 \rangle$	$\exists_{<}$: (2),(3)
(6)	clash	(4), (5)

Extending fuzzy $\mathcal{ALC}$

- Why - $\mathcal{ALC}$ is much less expressive than OWL Lite.
- Some extensions:
  - consider transitive roles + inverse roles ($f-\mathcal{SI}$), or transitive, inverse roles + role inclusion axioms + number restriction ($f-\mathcal{SHIN}$), etc : the tableau is similar to that for fuzzy $\mathcal{ALC}$.
  - consider concrete domains: new tableau.
- The need of concrete domains:
  - $\mathcal{I} \models \exists R.A$, where $\Delta^{\mathcal{I}} = \{a, 3\}$, $A^{\mathcal{I}} = \{3\}$, $R^{\mathcal{I}} = \{(a, 3)\}$.
  - consider $R : age$, $A :\geq 18$, $\mathcal{I} \not\models \exists age. \geq 18$ since $3 \not\geq 18$.
  - concrete domain $\mathrm{D} = \langle \Delta_{\mathrm{D}}, \Phi_{\mathrm{D}} \rangle$, Δ_{D}:interpretation domain, Φ_{D}: a set of *domain predicates* d with a **fixed** interpretation $d^{\mathrm{D}} : \Delta_{\mathrm{D}}^n \rightarrow [0, 1]$.



$$Minor = Person \sqcap \exists age. \leq 18$$
$$YoungPerson = Person \sqcap \exists age.Young$$

Fuzzy $\mathcal{ALC}(\mathrm{D})$

New constructors:

Syntax	Semantics
$C, D \rightarrow m(C) \mid$	$(m(C))^{\mathcal{I}}(x) = fm(C^{\mathcal{I}}(x))$
$\exists T.D \mid$	$(\exists T.D)^{\mathcal{I}}(x) = \sup_{o \in \Delta_{\mathrm{D}}} T^{\mathcal{I}}(x, o) \wedge D^{\mathcal{I}}(o)$
$\forall T.D$	$(\forall T.D)^{\mathcal{I}}(x) = \inf_{o \in \Delta_{\mathrm{D}}} T^{\mathcal{I}}(x, o) \rightarrow D^{\mathcal{I}}(o)$

where m - modifiers

Modifiers:

- e.g. very, more-or-less,etc.
- change the membership functions, e.g. $very(C(x)) = C(x)^2$
- $SportCar = Car \sqcap \exists speed.very(High)$.

Assertions: $\langle a : C, n\rangle$, $a \approx b$, $a \not\approx b$

$\mathcal{I} \models \langle a : C, n\rangle$ iff $C^{\mathcal{I}}(a^{\mathcal{I}}) \geq n$ (similarly for roles).

Decision problems: similarly to those in fuzzy $\mathcal{ALC}$ + degrees of subsumption:

- $\mathcal{K} \models \langle A \sqsubseteq B, n\rangle$ iff for every model $\mathcal{I}$ of $\mathcal{K}$, $[\inf_{x \in \Delta^{\mathcal{I}}} A^{\mathcal{I}}(x) \rightarrow B^{\mathcal{I}}(x)] \geq n$.

New tableau for fuzzy $\mathcal{ALC}(\mathrm{D})$

- The tableau for fuzzy $\mathcal{ALC}$ does not work here
  - $\mathcal{K} = \{\langle a : \exists age.Young \geq 0.7\rangle, \langle a : \forall age.\neg Young < 1\rangle\}$.
  - from $\langle a : \exists age.Young \geq 0.7\rangle$, the only thing we know is "a has an age x and $x \leq 16$", i.e. $\langle (a, x) : age, 1\rangle, x \leq 16$.
  - then, $\mathcal{K}$ is satisfiable iff $10 < x \leq 16$.
- **New tableau:**
  - uses bounded Mixed Integer Program (bMIP) oracle.
    The general MIP is, given A, B: integer matrices, h: integer vector, find
    $$\bar{x} \in \mathbb{Q}^k, \bar{y} \in \mathbb{Z}^m$$
    $$f(\bar{x}, \bar{y}) = \min\{f(x, y) \mid Ax + By \geq h\}$$
  - works with Zadeh fuzzy logic, Lukasiewicz fuzzy logic, modifiers and concrete predicates are combinations of linear functions.
  - For BTVB problem:
    $$glb(\mathcal{K}, a : C) = \min\{x \mid \mathcal{K} \cup \{\langle a : \neg C, \neg x\rangle\} \text{ satisfiable}\}$$
    $$glb(\mathcal{K}, C \sqsubseteq D) = \min\{x \mid \mathcal{K} \cup \{\langle a : C \sqcap \neg D, \neg x\rangle\} \text{ satisfiable}\}.$$
    - Apply tableaux calculus, then call bMIP oracle.

Tableau rules

	If	Then
RA	$\langle \alpha, l\rangle \in S_i$	$S_{i+1} = S_i \cup \{x_\alpha \geq l\}$
R$\sqcap$	$\langle a : C \sqcap D, l\rangle \in S_i$	$S_{i+1} = S_i \cup \{\langle a : C, l\rangle, \langle a : D, l\rangle\}$
R$\sqcup$	$\langle a : C \sqcup D, l\rangle \in S_i$	$S_{i+1} = S_i \cup \{\langle a : C, x_1\rangle, \langle a : D, x_2\rangle, x_1 + x_2 = l,$ $x_1 \leq y, x_2 \leq 1 - y, x_i \in [0,1], y \in \{0,1\}\}$ where x_i is a new variable, y is a new control variable.
R$\exists$	$\langle a : \exists R.C, l\rangle \in S_i$	$S_{i+1} = S_i \cup \{\langle (a,b) : R, l\rangle, \langle b : C, l\rangle\}$ where b is a new abstract individual.
...	...	...

Example.

- $\mathcal{K} = \{C = A \sqcap B, \langle a : A, 0.3\rangle, \langle a : B, 0.4\rangle\}$.
- determine $glb(\mathcal{K}, a : C) = \min\{x \mid \mathcal{K} \cup \{\langle a : \neg C, \neg x\rangle\}$ satisfiable$\}$.
- after preprocessed, $S_0 = \{\langle a : A, 0.3\rangle, \langle a : B, 0.4\rangle, \langle a : \neg A \sqcup \neg B, 1 - x\rangle\}$.

	Constraint set	Rule
0	S_0	
1	$S_0 \cup \{\langle a : \neg A, x_1\rangle, \langle a : \neg B, x_2\rangle,$ $x_1 + x_2 = 1 - x,$ $x_1 \leq 1 - y, x_2 \leq y,$ $x_1, x_2 \in [0,1], y \in \{0,1\}\}$	**R**$\sqcup$
2	$S_1 \cup \{\langle x_{a:A} \leq 1 - x_1\rangle,$ $\langle x_{a:B} \leq 1 - x_2\rangle\}$	**R**$\bar{A}$ twice
3	$S_2 \cup \{\langle x_{a:A} \geq 0.3\rangle, \langle x_{a:B} \geq 0.4\rangle\}$	**R**A twice
4	Find $\min\{x \mid S_3\}$	bMIP
5	bMIP oracle: $x = 0.3$	

Therefore, $glb(\mathcal{K}, a : C) = 0.3$.

Towards fuzzy OWL DL

Recall: OWL DL is equivalent to $\mathcal{SHOIN}(\mathrm{D})$.

New constructors:

Syntax	Semantics	
$\geq nS$ |	$(\geq nS)^{\mathcal{I}}(x)$	$= \sup_{y_1,\ldots,y_n \in \Delta^{\mathcal{I}}} \bigwedge_{i=1}^{n} S^{\mathcal{I}}(x, y_i)$
$\leq nS$ |	$(\leq nS)^{\mathcal{I}}(x)$	$= \neg(\geq n+1S)^{\mathcal{I}}(x)$
$\{a_1, \ldots, a_n\}$ |	$\{a_1, \ldots, a_n\}^{\mathcal{I}}(x)$	$= \bigvee_{i=1}^{n} a_i^{\mathcal{I}} = x$
$\{c_1, \ldots, c_n\}$ |	$\{c_1, \ldots, c_n\}^{\mathcal{I}}(o)$	$= \bigvee_{i=1}^{n} c_i^{\mathcal{I}} = o$
S^-	$(S^-)^{\mathcal{I}}(x, y)$	$= S^{\mathcal{I}}(y, x)$

- Knowledgebase = ABox + TBox + RBox, $\mathcal{K} = \langle \mathcal{T}, \mathcal{R}, \mathcal{A} \rangle$
- A RBox:
  - transitivity axioms $trans(R)$
  - fuzzy role inclusion axioms of the form $\langle \alpha \circ n \rangle$, where α is a role inclusion axiom.
- Assertions, terminological axioms, satisfiability, and decision problems are similar to fuzzy $\mathcal{ALC}$.
- However, at the moment there is no calculus for decision problems in fuzzy $\mathcal{SHOIN}(\mathrm{D})$ yet.

Conclusions & Outlook discussion

- Conclusions:
  - it is neccessary to extend classical DL towards the representation and reasoning with vague concepts.
  - this talk: an approach to fuzzy OWL DL: fuzzy $\mathcal{ALC}$, fuzzy $\mathcal{ALC}(\mathrm{D})$, and fuzzy $\mathcal{SHOIN}(\mathrm{D})$.
  - calculi for fuzzy $\mathcal{ALC}$, fuzzy $\mathcal{ALC}(\mathrm{D})$: available, but for fuzzy $\mathcal{SHOIN}(\mathrm{D})$: not yet available.
- Outlook:
  - Stoilos et al work on $f-\mathcal{SI}$, $f-\mathcal{SHIN}$, $f-\mathcal{SHOIN}(\mathcal{G})$ without considering concrete domains: calculus is similar to that in fuzzy $\mathcal{ALC}$.
  - [Straccia 2004] - A satisfiability-preserving transformation from fuzzy $\mathcal{ALC}$ to classical $\mathcal{ALCH}$: decision problems in fuzzy $\mathcal{ALC}$ can be solved using available reasoners for $\mathcal{ALCH}$.
  - [Sanchez & Tettamanzi 2004] proposes fuzzy quantifiers, e.g. $TopCustomer = Customer \sqcap (Usually) buy.ExpensiveItem$.